This pedagogical note introduces the accounting-based variance decomposition methodology of Vuolteenaho (2002) in a relatively simple format for the edification of accounting scholars and doctoral students who wish to use variance decomposition in their research. In addition to presenting an example that explicates the variance decomposition approach, we provide well-documented SAS and STATA programs for estimating variance decompositions from cross-sectional time-series data.

1. Introduction

The purpose of this note is to introduce the accounting-based variance decomposition methodology of Vuolteenaho (2002) in a relatively simple format for the edification of accounting scholars and doctoral students who wish to use variance decomposition in their research. In addition to presenting a simple example that explicates the variance decomposition approach, we provide potential users with well-documented SAS and STATA programs for estimating variance decompositions from cross-sectional time-series data. The essential importance of variance decomposition analysis to accounting research is that it provides an alternative (variance-based) approach to measuring “value relevance” that is complementary to the standard “value relevance” earnings response coefficient (ERC) workhorse. Readers interested in the variance decomposition “philosophy” and its applications to accounting research are referred to
the synthesis study by Callen (2009). The current note focuses on “how to” rather than on “why.”

2. Some Definitions

The Vuolteenaho (2002) variance decomposition model decomposes the variance of unexpected returns into the variance of earnings news, the variance of discount rate news—defined formally below—and their covariances. The model assumes that the accounting clean surplus relation holds. A formal proof of the model is included in Appendix A. The equation upon which the variance decomposition is based takes the form:

\[ r_t - E_{t-1}(r_t) = Ne_t - Nr_t \]  

where \( r_t \) denotes the (log) cum dividend stock return at time \( t \) and \( E_{t-1}(r_t) \) denotes the market’s expectation at time \( t - 1 \) of the stock return at time \( t \). Thus, the dependent variable \( (r_t - E_{t-1}(r_t)) \) is the unexpected stock return from period \( t - 1 \) to \( t \). \( Ne_t \) denotes earnings news at time \( t \) defined as the market’s revision from period \( t - 1 \) to \( t \) of (the appropriately discounted sum of) expected future earnings over the lifetime of the firm. For example, earnings news could be the market’s revision of the discounted sum of period \( t \) and period \( t + 1 \) expected earnings, provided no other future earnings are expected to change. \( Nr_t \) denotes discount rate news at time \( t \) defined as the market’s revision from period \( t - 1 \) to \( t \) of (the appropriately discounted sum of) expected future discount rates (expected future returns) over the lifetime of the firm.

Eq. (1) is essentially an identity with the following intuitive interpretation. Because the price of a security is the present value of expected future dividends, a revision to returns has only two potential sources, revisions to expected future dividends—measured here by earnings because of the clean surplus relation—and revisions to expected future discount rates, over the lifetime of the firm. Eq. (1) indicates that a positive shock to expected future earnings results in a positive shock to returns whereas a positive shock to expected future discount rates results in a negative shock to returns, just as an increase in the expected yield reduces bond prices.

Formally, earnings news is defined as follows:

\[ N e_t = \Delta E_t \sum_{j=0}^{\infty} \rho^j \text{roe}_{t+j} \]  

1. Other accounting studies that use the variance decomposition methodology include Bhat (2008), Callen and Segal (2004), Callen, Hope, and Segal (2005), Callen, Livnat, and Segal (2006), and Callen, Segal, and Hope (2010).

2. It is not necessary to read the appendix to understand the remainder of this note.

3. Formally, \( r_t = \log[1 + (\Delta MV_t + D_t)/MV_{t-1}] \) where \( MV_t \) is market value of equity at time \( t \), \( D_t \) total cash dividends at time \( t \), and \( \Delta MV_t = MV_t - MV_{t-1} \). We follow the common convention in this literature in denoting variables by uppercase letters and their log by lower case letters.
where $\Delta E_t = E_t - E_{t-1}$ denotes the revision from period $t - 1$ to period $t$, $\rho$ is a discount rate and $\text{roe}_t$ is the (log) return on book value equity at time $t$.\(^4\)

Note that earnings news can be decomposed into the conventional earnings surprise $\Delta E_t \text{roe}_t$ plus the revision to future earnings $\Delta E_t \sum_{j=1}^{\infty} \rho^j \text{roe}_{t+j}$. Thus earnings news generalizes the notion of an earnings surprise to the entire gamut of future earnings over the firm’s lifetime.

Similarly, discount rate news is defined formally as:

$$ N_{rt} = \Delta E_t \sum_{j=1}^{\infty} \rho^j r_{t+j} \quad (3) $$

The variance decomposition is obtained by taking variances across eq. (1) to yield the following:\(^5\)

$$ \text{VAR}(r_t - E_{t-1}(r_t)) = \text{VAR}(N_{et}) + \text{VAR}(N_{rt}) - 2\text{COV}(N_{et}, N_{rt}) \quad (4) $$

### 3. Computing Unexpected Returns, Earnings News, and Discount Rate News

Estimating the components of eqs. (1) and (4) requires, in turn, estimates of expectations over the lifetime of the firm. These are obtained by assuming a system of log-linear dynamic equations for market returns, return on equity and any other variables assumed to affect market returns and return on equity. This is similar to the Ohlson (1995) and Feltham-Ohlson (1995, 1996) models in which linear dynamics are assumed for the same reason—namely, estimating expectations of the variables of interest over time. Like Ohlson (1995) and Feltham-Ohlson (1995, 1996), the dynamic is assumed to have an autoregressive structure but, unlike their equations, we allow for all equations to be interrelated in a multivariate Vector Autoregressive (VAR) system.\(^6\) To simplify the discussion, we assume the simplest VAR structure possible for an accounting-based analysis, which takes the form:\(^7\)

$$ r_t = \alpha_1 r_{t-1} + \alpha_2 \text{roe}_{t-1} + \eta_{1t} \quad (5a) $$

\(^4\) Formally, $\text{roe}_t = \log(1 + X_t / \text{BV}_{t-1})$ where $X_t$ is earnings in period $t$ and $\text{BV}_t$ is book value of equity at period $t$. Note that the definition of the variables in this note are given by the Vuolteenaho model (2002) and are not arbitrary. Thus, the division by prior book value equity in the above definition (as opposed say to prior period market value) is model driven.

\(^5\) The terms on the right-hand side of eq. (4) are often called variance contributions in the literature in that they contribute to the variance of unexpected returns.

\(^6\) For the implications of this, see Callen (2009).

\(^7\) All variables are assumed to be mean adjusted in this formulation.
\[
\text{roe}_t = \beta_1 r_{t-1} + \beta_2 \text{roe}_{t-1} + \eta_{1t}
\]

(5b)

where \(\eta_{1t}\) and \(\eta_{2t}\) are mean-zero error terms.

We will now compute the revision to returns \((r_t - E_{t-1}(r_t))\), earnings news \((Ne_t)\), and discount rate news \((Nr_t)\) for the VAR system of eqs. (5a) and (5b). Because the computations are simplified by matrix notation, we reformulate eqs. (5a) and (5b) as follows:

\[
z_t = \Gamma z_{t-1} + \eta_t
\]

(6)

where

\[
z_t = \begin{pmatrix} r_t \\ \text{roe}_t \end{pmatrix}, \hspace{1cm} \Gamma = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{pmatrix} \hspace{1cm} \text{and} \hspace{1cm} \eta_t = \begin{pmatrix} \eta_{1t} \\ \eta_{2t} \end{pmatrix}.
\]

As shown by Campbell and Shiller (1988a, 1988b) and Vuolteenaho (2002), earnings news and discount rate news can be computed as follows:

\[
Ne_t = (e_1 + \lambda_1)' \eta_t
\]

(7)

and

\[
Nr_t = z_t' \eta_t
\]

(8)

where \(\cdot\) denotes the transpose operator, \(e_k' = (0, \ldots, 1, \ldots, 0)\) is a row vector with one as the \(k\)th element, and zero elsewhere and

\[
z_k' = e_k' \rho \Gamma (I - \rho \Gamma)^{-1}
\]

(9)

The term \((I - \rho \Gamma)^{-1}\) is the matrix equivalent of the present value of a sum.\(^8\)

As in Vuolteenaho (2002), eqs. (7) through (9) presuppose that discount rate news is computed directly and that earnings news is computed residually, by subtracting discount rate news from unexpected returns. In Section 4, we will explore other options.

We now are ready to compute the components of eqs. (1) and (4) for the VAR system of eqs. (5a) and (5b). It follows immediately from eq. (5a) that unexpected returns can be expressed as the error term:

\[
r_t - E_{t-1}(r_t) = \eta_{1t}
\]

(10)

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\(^8\) Eq. (8) be can proved fairly easily and similarly eq. (7). Specifically, it follows from eq. (6) that \(E_t(\rho r_{t+1}) - E_{t-1}(\rho r_{t+1}) = \rho E_t(z_{t+1})\) so that \(E_t(\rho r_{t+1}) - E_{t-1}(\rho r_{t+1}) = \rho E_t(z_{t+1}).\) Similarly, \(E_t(\rho^2 r_{t+2}) = \rho^2 E_t(GE(z_{t+1})) = \rho^2 e_1' \Gamma^2 z_{t+1}\) and \(E_{t-1}(\rho^2 r_{t+2}) = \rho^2 e_1' \Gamma^2 z_{t+1}\) so that \(E_t(\rho^2 r_{t+2}) - E_{t-1}(\rho^2 r_{t+2}) = \rho^2 e_1' \Gamma^2 z_{t+1}\) and so on ad infinitum. Thus, \(\Delta E_t = \sum_{j=1}^{\infty} (E_t(\rho^j r_{t+j}) - E_{t-1}(\rho^j r_{t+j})) = e_1' \rho \Gamma y_1 + e_1' \rho^2 \Gamma^2 y_2 + \ldots + e_1' \rho^N \Gamma^N y_N + \ldots = e_1' \rho \Gamma (I - \rho \Gamma)^{-1} y_1.\)
In a slightly more complicated fashion, earnings news and discount rate news can be expressed as simple linear functions of the error terms:

\[ k_1 \eta_{1t} + k_2 \eta_{2t} \]

where the \( k_i \) are functions of the parameters of \( \Gamma \). To see this, substitute eqs. (5a) and (5b) into eq. (7) to yield the following:

\[
Net_t = (e_1 + \hat{z}_1) \eta_t
= e'_1 \left[ I + \rho \Gamma (I - \rho \Gamma)^{-1} \right] \eta_t
= e'_1 (I - \rho \Gamma)^{-1} \eta_t
= (1, 0) \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) - \rho \left( \begin{array}{cc} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{array} \right)^{-1} \left( \begin{array}{c} \eta_{1t} \\ \eta_{2t} \end{array} \right)
= (1, 0) \left( \begin{array}{cc} 1 - \rho \alpha_1 & -\rho \alpha_2 \\ -\rho \beta_1 & 1 - \rho \beta_2 \end{array} \right)^{-1} \left( \begin{array}{c} \eta_{1t} \\ \eta_{2t} \end{array} \right)
= 1 \frac{1}{H} (1, 0) \left( \begin{array}{cc} 1 - \rho \beta_2 & \rho \beta_2 \\ \rho \beta_1 & 1 - \rho \alpha_1 \end{array} \right) \left( \begin{array}{c} \eta_{1t} \\ \eta_{2t} \end{array} \right)
= \frac{(1 - \rho \beta_2)}{H} \eta_{1t} + \frac{\rho \beta_2}{H} \eta_{2t}
\]

(11)

where \( H = (1 - \rho \alpha_1)(1 - \rho \beta_2) - \rho^2 \alpha_2 \beta_1 \). Similarly,

\[
Nr_t = \chi' \eta_t
= e'_1 \rho \Gamma (I - \rho \Gamma)^{-1} \eta_t
= (1, 0) \rho \left( \begin{array}{cc} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{array} \right) \left[ \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right] - \rho \left( \begin{array}{cc} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{array} \right)^{-1} \left( \begin{array}{c} \eta_{1t} \\ \eta_{2t} \end{array} \right)
= \frac{1}{H} (\rho \alpha_1 (1 - \rho \beta_2) + \rho^2 \alpha_2 \beta_1) + \rho \alpha_2 (1 - \rho \alpha_1) + \rho^2 \alpha_1 \beta_2 \left( \begin{array}{c} \eta_{1t} \\ \eta_{2t} \end{array} \right)
= \frac{(\rho \alpha_1 (1 - \rho \beta_2) + \rho^2 \alpha_2 \beta_1)}{H} \eta_{1t} + \frac{(\rho \alpha_2 (1 - \rho \alpha_1) + \rho^2 \alpha_1 \beta_2)}{H} \eta_{2t}
\]

(12)

To check these calculations, note that subtracting eq. (12) from eq. (11) yields eq. (1), \( Net_t - Nr_t = \eta_{1t} = r_t - E_{t-1}(r_t) \), as required.

Eqs. (10), (11), and (12) yield the components of eq. (1) provided that the parameters of eqs. (5a) and (5b) and the errors \( \eta_{1t} \) and \( \eta_{2t} \) are known. In general, this is not the case, and the parameters and error terms have to be estimated. We discuss estimation in Section 4. In this section, we assume
that we have the relevant estimates from estimating eqs. (5a) and (5b). Let $\hat{Ne}_t$ and $\hat{Nr}_t$ denote the estimated earnings news and discount rate news, respectively. These estimated news items are obtained by replacing the parameters $\alpha_k$ and $\beta_k$ in eqs. (10), (11), and (12) by their estimates $\hat{\alpha}_k$ and $\hat{\beta}_k$ and the error terms $\eta_{it}$ by their residual equivalent $\hat{\eta}_{it}$. Otherwise, the formulas are the same.

We now show the computation of the (sample) variances of eq. (4) for our example [eqs. (5a) and (5b)] assuming that we have the relevant estimates. Specifically, let $\sum = \frac{1}{n} (\hat{\eta}_t \hat{\eta}_t')$ denote the estimated Variance–Covariance matrix of the residuals. Let $\hat{\lambda}_k$ be identical to $\lambda_k$—see eq. (9)—with the parameters of $\Gamma$ replaced by the parameter estimates. Following Campbell and Schiller (1998a, 1998b) and Vuolteenaho (2002), the variances in eq. (4) can be estimated by the following:

\[
VAR(r_t - E_{t-1}(r_t)) = \frac{1}{n} \hat{\eta}_{1t}^2,
\]

(13a)

\[
VAR(\hat{Ne}_t) = \text{VAR}\left[\left(e_1 + \hat{\lambda}_1\right) \hat{\eta}_t\right] = \frac{1}{n} \left(e_1 + \hat{\lambda}_1\right)' \hat{\eta}_t \hat{\eta}_t' \left(e_1 + \hat{\lambda}_1\right) = \frac{1}{n} \hat{Ne}_t^2
\]

(13b)

\[
VAR(\hat{Nr}_t) = \text{var}\left(\hat{\lambda}_1 \hat{\eta}_t\right) = \frac{1}{n} \hat{\eta}_t \hat{\eta}_t' \hat{\lambda}_1 = \frac{1}{n} \hat{Nr}_t^2
\]

(13c)

\[
\text{COVAR}(\hat{Ne}_t, \hat{Nr}_t) = \hat{\lambda}_1 \sum (e_1 + \hat{\lambda}_1) = \frac{1}{n} \hat{Nr}_t \hat{Ne}_t
\]

(13d)

The variances and the covariance contributions for the above example will take on the simple quadratic form:

\[
h_1 \hat{\eta}_{1t}^2 + h_2 \hat{\eta}_{1t} \hat{\eta}_{2t} + h_3 \hat{\eta}_{2t}^2
\]
where the $h_i$ are functions of the estimated parameters. More specifically,

$$VAR(\hat{N}_e) = \frac{1}{n} \hat{N}_e^2$$

$$= \left( 1 - \rho \hat{\rho}_2 \right)^2 \hat{\eta}_{1i}^2 + 2\rho \hat{\rho}_2 \left( 1 - \rho \hat{\rho}_2 \right) \hat{\eta}_{1i} \hat{\eta}_{2i} + \rho^2 \hat{\rho}_2^2 \hat{\eta}_{2i}^2$$  \hspace{1cm} (14a)

where $\hat{H} = (1 - \rho \hat{\alpha}_1)(1 - \rho \hat{\beta}_2) - \rho^2 \hat{\alpha}_2 \hat{\beta}_1$

$$VAR(\hat{N}_r) = \frac{1}{n} \hat{N}_r^2$$

$$= \left[ \rho \hat{\rho}_1 + \left( 1 - \rho \hat{\beta}_2 \right) + \rho^2 \hat{\rho}_2 \hat{\beta}_1 \right]^2 \hat{\eta}_{1i}^2$$

$$+ \frac{2\rho \hat{\rho}_2 \left( 1 - \rho \hat{\beta}_2 \right) + \rho^2 \hat{\rho}_2 \hat{\beta}_1 \left[ \rho \hat{\rho}_2 (1 - \rho \hat{\rho}_1) + \rho^2 \hat{\rho}_2 \hat{\beta}_2 \right]}{nH^2} \hat{\eta}_{1i} \hat{\eta}_{2i}$$

$$+ \left[ \rho \hat{\rho}_2 (1 - \rho \hat{\rho}_1) + \rho^2 \hat{\rho}_2 \hat{\beta}_2 \right]^2 \hat{\eta}_{2i}^2$$  \hspace{1cm} (14b)

$$COVAR(\hat{N}_e, \hat{N}_r) = \frac{1}{n} \hat{N}_r \hat{N}_e$$

$$= \left( 1 - \rho \hat{\rho}_2 \right) \left[ \rho \hat{\rho}_1 + \left( 1 - \rho \hat{\beta}_2 \right) + \rho^2 \hat{\rho}_2 \hat{\beta}_1 \right] \hat{\eta}_{1i}^2$$

$$+ \frac{\rho^3 \hat{\rho}_2^2 \hat{\beta}_1 + 2\rho^2 \hat{\rho}_1 \hat{\beta}_2 \left( 1 - \rho \hat{\beta}_2 \right) + \rho \hat{\rho}_2 (1 - \rho \hat{\rho}_1) \left( 1 - \rho \hat{\beta}_2 \right)}{nH^2} \hat{\eta}_{1i} \hat{\eta}_{2i}$$

$$+ \frac{\rho \hat{\rho}_2 \hat{\rho}_1 \hat{\rho}_2 (1 - \rho \hat{\rho}_1) + \rho^2 \hat{\rho}_1 \hat{\beta}_2 \hat{\rho}_2}{nH^2} \hat{\eta}_{2i}^2$$  \hspace{1cm} (14c)

### 4. Estimation Issues

Vuolteenaho (2002) estimates the variance decomposition of discount rate news and earnings news over the entire sample or over portfolios of firms. This approach allows for only a limited analysis of the cross-sectional variation of the news components. An alternative approach is to estimate the VAR at an aggregated level, but then compute the news components and their variances using firm-year-level residuals. For example, one can estimate each equation of the VAR system [e.g., eqs. (5a) and (5b)] separately either by panel data regression techniques or by weighted least squares for each industry. A common approach is to use the Fama-French (1997) industry classification. This will yield VAR parameters at the industry level, but the residuals will be at the firm-year level. Firm-year-level estimates of $\hat{N}_r$ and $\hat{N}_e$ and their variances can then be computed using the formulas above.

In the discussion of the example, discount rate news is computed directly as per eq. (8) and earnings news is computed residually as $r_i - \hat{E}_{i-1}(r_i) - \hat{N}_r$ to
obtain eq. (7). This approach guarantees the equality of eq. (1) by construction. An alternative approach estimates earnings news directly and discount rate news residually. In the latter case,

$$Ne_t = e'_2(I - \rho \Gamma)^{-1}$$

$$= \lambda'_2 \eta_t$$

and

$$Nr_t = (e'_2 + \lambda'_2) \eta_t$$

This approach also guarantees that eq. (1) holds by construction. However, the two approaches may well yield different estimates of the news items and their variances. In fact, one should anticipate that the news item that is estimated residually incorporates more of the error in the unexpected return relation than the news item that is estimated directly.\(^9\) A third approach estimates both earnings news and discount rate news directly, in which case, the equality of eq. (1) cannot be guaranteed.\(^10\)

For simplicity, the equations and the example above assumed that returns are measured gross of the risk-free rate. In empirical work, it is common to subtract (the log of one plus) the risk-free rate from returns, which requires that the risk-free rate be subtracted either from earnings news or discount rate news. If the former is true, then earnings news is defined as $\Delta E_t \sum_{j=0}^{\infty} \rho (\text{ro}e_{t+j} - i_{t+j})$ where $i_t = \log(1 + \text{risk} - \text{free rate})$.

$\rho$ is the average log market-to-book ratio and is estimated as the convex combination of the log dividend-to-book value and the log dividend-to-price ratios for the United States. Because the convex combination for the United States historically is approximately 4 percent, $\rho$ is often assumed empirically to take on a value of 0.967 as in Vuolteenaho (2002). However, the exact value of $\rho$ appears to have little impact on the results within the range between 0.95 and 1.

5. The Programs: General Issues

The SAS and STATA programs to implement a variance decomposition using cross-sectional time-series data are included in Appendix B. These programs assume that the VAR is of the following form:

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9. As shown in Appendix A, eq. (1) holds as an approximation.
10. In a study that evaluates the relative “value relevance” of earnings components, it is a good idea to measure the news items of all earnings components directly so that the earnings components are computed on the same basis.
\[ r_t = \alpha_1 r_{t-1} + \alpha_2 \text{roet}_{t-1} + \alpha_3 \text{bm}_{t-1} + \eta_{1t} \]  
(17a)

\[ \text{roet}_t = \beta_1 r_{t-1} + \beta_2 \text{roet}_{t-1} + \beta_3 \text{bm}_{t-1} + \eta_{2t} \]  
(17b)

\[ \text{bm}_t = \gamma_1 r_{t-1} + \gamma_2 \text{roet}_{t-1} + \gamma_3 \text{bm}_{t-1} + \eta_{3t} \]  
(17c)

where \( \text{bm}_t \) denotes the log book to market ratio at time \( t \).\(^{11}\)

Each equation is estimated by weighted least squares. These programs estimate earnings news residually and discount rate news directly as in Vuolteenaho (2002). The programs also indicate how to estimate earnings news directly. Because the news items are measured at the firm-year level, we set \( n = 1 \) in eqs. (13a) through (13d).

The programs estimate the VAR coefficient matrix, discount rate news (henceforth return news) and earning news, and the variance contributions of return news and earnings news at the firm-year level. To control for industry effects, the program estimates the variables for each industry separately using macros. The industries are defined according to Fama and French (1997). The initial data set, labeled “INIT”, requires the following variables:

1. Firm Identifier: The program uses \( \text{PERMNO} \).
2. Year: The program uses \( \text{YEARA} \).
3. Return variable: Labeled in the program as \( \text{LogR} \). \( \text{LogR} \) is computed as the log of one plus the annual cum dividend return minus the log of one plus the annualized three month Treasury bill rate.
4. Earnings variable: Labeled in the program \( \text{LogE} \). \( \text{LogE} \) is computed as the log of one plus \( \text{ROE} \) minus the log of one plus the annualized three-month Treasury bill rate. \( \text{ROE} \) is computed as income before extraordinary items (DATA18) scaled by beginning of period book value of equity (DATA60).
5. Book-to-Market variable: Labeled in the program \( \text{LogBM} \). \( \text{LogBM} \) is computed as log of the book-to-market ratio at year end.
6. Industry Identifier variable: Labeled in the program \( \text{IND} \). \( \text{IND} \) takes the values of one to \( T \) where \( T \) is the number of industries.

In addition, the program requires one lag of each of \( \text{LogR}, \text{LogE}, \) and \( \text{LogBM} \), labeled as \( \text{LogR1}, \text{LogE1}, \) and \( \text{LogBM1} \), respectively. Overall, \( \text{INIT} \) should include \( \text{PERMNO}, \text{YEARA}, \text{IND}, \text{LogR}, \text{LogE}, \text{LogBM}, \text{LogR1}, \text{LogE1}, \) and \( \text{LogBM1} \). One should remove the bottom and top 1 percent of each of \( \text{LogR}, \text{LogE}, \text{LogBM}, \text{LogR1}, \text{LogE1}, \) and \( \text{LogBM1} \) to mitigate the impact of outliers.

\(^{11}\) The accounting studies cited in footnote 1 use this three-equation VAR formulation.
REFERENCES


APPENDIX A


The Vuolteenaho (2002) return decomposition relation is derived from the definition of the log book-to-market ratio and the accounting clean surplus relation. Formally, define $BV_t$ to be the book value of equity at time $t$, $MV_t$ the market value of equity at time $t$, $D_t$ dividends at time $t$, $X_t$ earnings in period $t$, and $\Delta$ is the differencing operator.

From the book-to-market ratio definition, the following is obtained:

$$ \frac{BV_t}{MV_t} = \frac{1 + \Delta BV_t/BV_{t-1}}{1 + (\Delta MV_t + D_t)/MV_{t-1} - D_t/MV_{t-1}} \cdot \frac{BV_{t-1}}{MV_{t-1}} $$

where the second equality follows from the accounting clean surplus relation:

$$ \Delta BV_t = BV_t - BV_{t-1} = X_t - D_t $$

Taking logs of both sides of eq. (1) yields the log book-to-market ratio ($bmt$):

$$ bm_t = \log \left( \frac{BV_t}{MV_t} \right) = \log \left( \frac{1 + X_t/BV_{t-1} - D_t/BV_{t-1}}{1 + (\Delta MV_t + D_t)/MV_{t-1} - D_t/MV_{t-1}} \right) + bm_{t-1} $$

If $D_t = 0$, then eq. (A3) can be written in the convenient log-linear form:

$$ roe_t - rt = bm_t - bm_{t-1} $$

where $roe_t = \log(1 + X_t/BV_{t-1})$ = the log of (one plus deflated) earnings and $rt = \log(1 + (\Delta MV_t + D_t)/MV_{t-1})$ = the log of (one plus) the cum dividend equity return.

If $D_t \neq 0$, then eq. (A3) can no longer be expressed directly in log-linear form. To maintain the log-linearity property, Vuolteenaho (2002) applies a Taylor series approximation to eq. (A3)—expanding around a convex combination of the unconditional means of the log dividend-to-book value and the log dividend-to-price ratios—thereby yielding the log-linear pricing relation:
\[ \text{roet}_t - r_t = \rho \text{bm}_t - \rho \text{bm}_{t-1} + \xi_t \]  \hspace{1cm} (A5)

where \( \xi_t \) = approximation error and \( \rho \) = discount rate coefficient.

Iterating eq. (A5) forward \( N \) periods gives the following:

\[ \text{bm}_{t-1} = \sum_{j=0}^{N} \rho^j \text{roet}_{t+j} - \sum_{j=0}^{N} \rho^j \text{roet}_{t+j} + \sum_{j=0}^{N} \rho^j \xi_{t+j} + \rho^{N+1} \text{bm}_{t+N} \]  \hspace{1cm} (A6)

Assuming the book-to-market ratio follows a covariance stationary process, the variance of the last term of eq. (A6) converges to zero as the horizon \( N \) increases to infinity, yielding the following:

\[ \text{bm}_{t-1} = \sum_{j=0}^{\infty} \rho^j \text{roet}_{t+j} - \sum_{j=0}^{\infty} \rho^j \text{roet}_{t+j} + \sum_{j=0}^{\infty} \rho^j \xi_{t+j} \]  \hspace{1cm} (A7)

Taking the change in expectations from time period \( t-1 \) to \( t \) and assuming that the change in expectations of the cumulative (discounted) error approximation is sufficiently small gives the following:  \hspace{1cm} (A8)

\[ r_t - E_{t-1}(r_t) = \Delta E_t \sum_{j=0}^{\infty} \rho^j \text{roet}_{t+j} - \Delta E_t \sum_{j=1}^{\infty} \rho^j \text{roet}_{t+j} \]

where \( E_t \) = the expectations operator at time \( t \) and \( \Delta E_t(.) = E_t(.) - E_{t-1}(.) \). Eq. (A8) is the Vuolteenaho (2002) return decomposition relation. It can be expressed more succinctly as follows:

\[ r_t - E_{t-1}(r_t) = N e_t - N r_t \]  \hspace{1cm} (A9)

where \( N e_t = \Delta E_t \sum_{j=1}^{\infty} \rho^j \text{roet}_{t+j} = \text{Earnings News} \) and \( N r_t = \Delta E_t \sum_{j=1}^{\infty} \rho^j \text{roet}_{t+j} = \text{and Expected Return (Discount Rate) News} \).
The Program in STATA

The Programs

APPENDIX B

The Program in STATA

use INIT, clear
foreach v in LogE LogR LogBM LogEl LogR1 LogBM1 {
bysort ind: egen mn`v' = mean(`v')
gen dm`v' = `v' - mn`v'
}

*b compute the number of observations per year for each industry (nobs). 1/nobs will be used as the weight in the var regressions below, which are estimated using weighted least squares*
bysort ind yeara: egen float nobs = count (yeara)
gen w = 1/nobs

* create the vectors of residuals to be used in computing the news items. the missing values will be filled in later with the residuals from the var regressions below*
sort ind permno yeara
gen ER=.
gen EE=.
gen EB=.
save temp, replace

* estimate the var coefficient matrix for each industry excluding financials (sic 6000-6999), 44 industries in total.*

The macro does the following:

1. Estimates the var regressions by industry, one for each demeaned state variable (dmlogr, dmloge, dmlogbm) using weighted least squares.
2. The residuals from the dmlogr, dmloge, and dmlogbm regressions are saved in er, ee and eb, respectively. The program creates a new dataset, res.dta, which
3. The coefficients from each regression are saved into a new dataset, COF.DTA. At the end of the macro, COF.DTA should have 44 observations (as the number of industries) with 9 columns – the first three columns, COFDMLOGR1, COFDMLOGR2, and COFDMLOGR3 are the coefficients for DMLOGR1, DMLOGE1, and DMLOGBM1 from the return regression (i.e. where the dependent variable is DMLOGR). The next three columns, COFDMLOGE1, COFDMLOGE2, and COFDMLOGE3 are the coefficients for DMLOGR1, DMLOGE1, and DMLOGBM1 from the earnings regression (i.e. where the dependent variable is DMLOGE). The last three columns COFDMLOGBM1, COFDMLOGBM2, and COFDMLOGBM3 are the coefficients for DMLOGR1, DMLOGE1, and DMLOGBM1 from the book-to-market regression (i.e. where the dependent variable is DMLOGE). The data also include the industry identifier.

set more off
local i = 1
while `i' < 45{
    use temp, clear
    set more off
    display `"THIS IS THE"' `i' `"INDUSTRY OUT OF 45"'
    foreach v in dmLogR dmLogE dmLogBM{
        qui regress `v' dmLogR1 dmLogE1 dmLogBM1[ pweight=w] if ind==`i', ///
        noconstant
        predict e`v' if ind==`i', residual
        matrix cof`v' = e(b)
        svmat cof`v'
    }
    replace ER = edmLogR if ind==`i'
    replace EE = edmLogE if ind==`i'
    replace EB = edmLogBM if ind==`i'
    drop edm*
    save temp1, replace
    keep cof*
    gen ind==`i' if cofdmLogR1!=.
    keep if ind!=.
if `i' ==1{
    save COF, replace
} else{
    append using COF
    save COF, replace
}

use temp1, clear
drop cof*
save temp, replace
local i=`i'+1

use temp, replace
keep permno yeara ind ER EE EB
save RES, replace

/***************************/
/*COMPUTE THE LAMBDA''S*/
/***************************/
* THIS PART OF THE PROGRAM CREATES TWO VECTORS, LAMBDA1 AND LAMBDA2, FOR EACH INDUSTRY. LAMBDA1 IS USED TO COMPUTE RETURN NEWS DIRECTLY, AND LAMBDA2 IS USED TO COMPUTE EARNINGS NEWS INDIRECTLY.

local i=1
while `i'<=45{
    set more off
    * OBTAIN THE COEFFICIENTS FROM EACH REGRESSION FROM THE FILE COF.DTA AND TRANSFORM THEM INTO A ROW VECTOR. THE FIRST VECTOR INCLUDES THE COEFFICIENT FROM THE RETURN REGRESSION [COFDMLOGR1-COFDMLOGR3], THE SECOND FROM THE EARNINGS REGRESSION [COFDMLOGE1-COFDMLOGE3], AND THE THIRD FROM THE BOOK-TO-MARKET REGRESSION [COFDMLOGBM1-COFDMLOGBM3]. CREATE THE VAR COEFFICIENT MATRIX (A, 3X3) USING THE THREE VECTORS. IN ADDITION, CREATE e1 (=\([1,0,0]\)) AND e2 (=\([0,1,0]\)). ro IS THE DISCOUNT TERM = 0.967

    use COF, clear.
    keep if ind==`i'
    mkmat cofdmLogR1 cofdmLogR2 cofdmLogR3, matrix (row1)
**COMPUTE LAMBDAL1 (LABELED LAM1) AND LAMBDAL2 (LABELED LAM2).**

\[
\text{matrix lam1} = \text{e1} \cdot \text{ro} \cdot \text{A} \cdot \text{inv} (\text{I}(3) - \text{ro} \cdot \text{A}) \\
\text{matrix lam2} = \text{e1} + \text{lam1}
\]

**TO COMPUTE EARNINGS NEWS DIRECTLY REPLACE THE LINE ABOVE WITH**

\[
\text{matrix lam2} = \text{e2} \cdot \text{inv} (\text{I}(3) - \text{ro} \cdot \text{A})
\]

**TRANSFER THE LAMBDAL1 AND LAMBDAL2 VECTORS TO A DATASET NAMED LAM.DTA. THE PROGRAM USES LAM WHEN COMPUTING THE NEWS VARIABLES IN THE NEXT STEP. AT THE END OF THE MACRO, LAM.DTA HAS 44 ROWS WITH 7 COLUMNS. THE FIRST COLUMN INCLUDES THE INDUSTRY VARIABLE. COLUMNS TWO TO FOUR ARE THE COMPONENTS OF LAMBDAL1 AND ARE LABELED LAM11, LAM12 AND LAM13. COLUMNS FIVE TO SEVEN ARE THE COMPONENTS OF LAMBDAL2 AND ARE LABELED LAM21, LAM22 AND LAM32.**

\[
\text{svmat lam1} \\
\text{svmat lam2} \\
\text{keep ind lam*} \\
\text{if 'i' ==1{} \\
\text{save LAM, replace} \\
\text{} \\
\text{else{} \\
\text{append using LAM} \\
\text{save LAM, replace} \\
\text{}} \\
\text{local i='i'+1} \\
\text{}}
\]

//**********************************************************************************
//*COMPUTE THE NEWS ITEMS USING THE RESIDUALS AND THE LAMBDAS*/
//**********************************************************************************

**THIS PART OF THE PROGRAM COMPUTES THE NEWS ITEMS AND THEIR VARIANCE CONTRIBUTIONS USING THE LAMBDAS (SAVED IN**
(LAM.DTA) AND THE RESIDUALS FROM THE REGRESSIONS (SAVED IN RES.DTA). THE COMPUTATION PROCEDURE IS DONE SEPARATELY FOR EACH INDUSTRY. THE PROCEDURE USES THE MATA LANGUAGE, WHICH IS AVAILABLE FOR STATA VERSION 9 AND UP. THE NEWS ITEMS AND VARIANCES ARE SAVED TO NEWS.DTA

*CREATE THE LAMBDA1 AND LAMBDA2 VECTORS

local i=1
while ‘i’<45{
set more off
use LAM, clear
keep if ind==‘i’
mkmat lam11 lam12 lam13, matrix (lam1)
mkmat lam21 lam22 lam23, matrix (lam2)
}

* CREATE A MATRIX FROM THE THREE VECTORS OF RESIDUALS: ER, EE AND EB, IN THAT ORDER. THE MATRIX DIMENSIONS ARE KX3 WHERE K IS THE NUMBER OF OBSERVATIONS IN THE INDUSTRY. IN ADDITION, READ INTO MATA THE LAMBDA1 AND LAMBDA2 VECTORS, LABELED LAM1 AND LAM2, RESPECTIVELY.

use RES, clear
keep if ind==‘i’
mata: res=st_data(.,(‘ER’’, ‘EE’’, ‘EB’’))
mata: lam1=st_matrix(‘’lam1’’)
mata: lam2=st_matrix(‘’lam2’’)

* COMPUTE THE NEWS ITEMS AND VARIANCE CONTRIBUTIONS. DISCOUNT RATE NEWS AND EARNINGS NEWS ARE LABELED Nr AND Ne, RESPECTIVELY. THE VARIANCE CONTRIBUTION OF RETURNS AND THE VARIANCE CONTRIBUTION OF EARNINGS ARE LABELED Vr AND Ve, RESPECTIVELY. THE COVARIANCE BETWEEN DISCOUNT RATE NEWS AND EARNINGS NEWS IS LABELED COV_re. Nr=LAMBDA1*RES’, Ne=LAMBDA2*RES’, Vr=NR*Nr, Ve=Ne*Ne, COV_re=Nr*Ne.

mata: Nr=(lam1*res’)’
mata: Ne=(lam2*res’)’
mata: idx=st_addvar(‘’float’’, ‘’Nr’’)
mata: st_store(.,idx,Nr)
mata: idx1=st_addvar(‘’float’’, ‘’Ne’’)
mata: st_store(.,idx1,Ne)
if ind==1 {
save NEWS, replace
} else {
append using NEWS
save NEWS, replace
}
local i='i'+1
}

use NEWS, clear
gen Vr=Nr*Nr
gen Ve=Ne*Ne
gen COV_re=Ne*Nr
save NEWS, replace

**The Program in SAS**

libname vdc 'c:\vdc';
run;

/*DEMEANING THE VARIABLES*/

data a1; set vdc.INIT (keep=PERMNO YEARA IND LogR LogE LogBM LogR1 LogE1 LogBM1);
proc sort; by IND;
proc means noprint; by IND;
var LogR LogE LogBM LogR1 LogE1 LogBM1;
output out=z mean=mnr mne mnbm mnr1 mne1 mnbm1;
run;

data a2; merge a1 z; by IND;
dmLogR=LogR-mnr;
dmLogR1=LogR1-mnr1;
dmLogBM=LogBM-mnbm;
dmLogBM1=LogBM1-mnbm1;
dmLogE=LogE-mne;
dmLogE1=LogE1-mne1;
keep PERMNO YEARA IND dm:;
run;
/*COMPUTE THE NUMBER OF OBSERVATIONS PER YEAR FOR EACH INDUSTRY (NOBS). 1/NOBS WILL BE USED AS THE WEIGHT IN THE VAR REGRESSIONS BELOW, WHICH ARE ESTIMATED USING WEIGHTED LEAST SQUARE*/

```sas
data a2; set a2;
proc sort; by IND YEARA;
proc means noprint; by IND YEARA;
var YEARA;
output out=x1 n=nobs;
run;

data a3; merge a2 x1 (keep=IND YEARA nobs); by IND YEARA;
w=1/nobs
run;

/*
VAR COEFFICIENT MATRIX AND RESIDUALS
**********************************************************************
1. ESTIMATE THE VAR REGRESSIONS BY INDUSTRY, ONE FOR EACH
DEMEANED STATE VARIABLE (DMLOGR, DMLOGE, DMLOGBM) USING
WEIGHTED LEAST SQUARE.
2. THE RESIDUALS FROM THE DMLOGR, DMLOGE, AND DMLOGBM
REGRESSIONS ARE SAVED IN ER, EE AND EB, RESPECTIVELY. THE
PROGRAM CREATES A NEW (PERMANENT) DATASET, RES, WHICH
INCLUDES PERMNO, YEARA, IND, ER, EE, AND EB. RES HAS AS
MANY OBSERVATIONS AS INIT.
3. THE COEFFICIENTS FROM EACH REGRESSION ARE SAVED INTO A
NEW (PERMANENT) DATASET, COF. FOR EACH INDUSTRY, COF
CONTAINS THE COEFFICIENTS OF DMLOGR1, DMLOGE1, DMLOGBM1
FROM EACH OF THE THREE REGRESSIONS.
*/

data a3; set a3;
proc sort; by IND;
proc reg noprint edf outest=cof; by IND;
model dmLogR=dmLogR1 dmLogE1 dmLogBM1/noint;
weight w;
output out=u1 r=ER;
model dmLogE=dmLogR1 dmLogE1 dmLogBM1/noint;
weight w;
output out=u2 r=EE;
model dmLogBM=dmLogR1 dmLogE1 dmLogBM1/noint;
weight w;
```
output out=u3 r=EB;
data vdc.cof; set cof (keep=_depvar_ IND dmLogR1 dmLogE1 dmLogBM1);
run;
proc sort data=u1 (keep=PERMNO YEARA IND ER); by PERMNO YEARA;
proc sort data=u2 (keep=PERMNO YEARA EE); by PERMNO YEARA;
proc sort data=u3 (keep=PERMNO YEARA EB); by PERMNO YEARA;
data vdc.res; merge u1 u2 u3; by PERMNO YEARA;
run;
/*
COMPUTE THE NEWS USING THE RESIDUALS AND THE VAR
COEFFICIENTS
******************************************************************************
THE COMPUTATION PROCEDURE IS DONE SEPARATELY FOR EACH INDUSTRY USING A MACRO.
THE PROGRAM READS THE VECTORS OF RESIDUALS INTO A MATRIX CALLED RES_MAT AND THE VAR COEFFICIENT MATRIX INTO COF_MAT.
IN THE FIRST STAGE, LAMBDAS1 AND LAMBDASON, LABELED LAM1 AND LAM2, RESPECTIVELY, ARE COMPUTED USING THE VAR COEFFICIENT MATRIX. LAM1 AND LAM2 ARE A 1X3 VECTORS.
RETURN (EARNINGS) NEWS ARE THEN OBTAINED BY MULTIPLYING LAM1 (LAM2) BY RES_MAT' (3XK, K = NUMBER OF OBSERVATIONS IN THE INDUSTRY)
THE NEWS ITEMS ARE THEN CONVERTED INTO A DATASET CALLED X1. AT THE END OF THE MACRO A PERMANENT DATA SET CALLED NEWS IS CREATED WITH NR AND NE FOR EACH FIRM YEAR. THE VARIANCE CONTRIBUTION OF RETURN NEWS (Vr) AND THE VARIANCE CONTRIBUTION OF EARNINGS NEWS (Ve) AS WELL AS THE COVARIANCE BETWEEN EARNINGS AND DISCOUNT RATE NEWS (COV_re) ARE COMPUTED USING Nr AND Ne. SPECIFICALLY, Vr=Nr^2, Ve=Ne^2, COV_re=Nr*Ne.
*/
proc datasets;
delete x x1;
run;
/*THIS MACRO COMPUTES THE NEWS FOR EACH INDUSTRY SEPERATELY*/

%macro myloop;
%do j=1 %to 44;

data u; set vdc.res;
if IND=&j;
data g; set vdc.cof;
if IND=&j;
run;
proc iml;

/*READ THE VECTORS OF RESIDUALS INTO A MATRIX CALLED RES_MAT, AND THE VAR COEFFICIENT MATRIX INTO COF_MAT. THE MATRIX ID_MAT KEEPS THE OBSERVATION ORDER*/

use u;
read all var {er ee eb} into res_mat;
read all var {PERMNO YEARA} into id_mat;
use g;
read all var {dmLogR1 dmLogE1 dmLogBM1} into cof_mat;

e1={1 0 0};
e2={0 1 0};
ro=0.967;

/*USING RES_MAT AND COF_MAT, COMPUTE LAMBDA1 (LAM1) AND LAMBDA2 (LAM2), AND DISCOUNT RATE NEWS (NR) AND EARNINGS NEWS (NE)*/
lam1=e1*ro*cof_mat*inv(I(3)-ro*cof_mat);
lam2=e1+lam1

/*TO COMPUTE EARNINGS NEWS DIRECTLY, REPLACE THE LINE ABOVE WITH lam2=e2*inv(I(3)-ro*cof_mat);*/
Nr=(lam1*res_mat');
Ne=(lam2*res_mat');

/*CREATE THE NEWS MATRIX FROM THE NR AND NE VECTORS, AND TRANSFER IT INTO A DATASET CALLED X. THE DATASET X CONTAINS
THE OBSERVATION ID’S AS WELL AS THE NEWS ITEMS FOR THE INDUSTRY*/

\[ z = \text{id\_mat} | \text{Nr} | \text{Ne}; \]
create x from z;
append from z;
quit;
data x; set x;
IND=&j;
run;

/*APPEND THE INDUSTRY INFORMATION IN X INTO A DATASET CALLED X1 WHICH CONTAIN THE NEWS ITEMS FOR ALL INDUSTRIES*/

proc datasets library=work;
append base=work.x1 data=work.x force;
%end;
%mend myloop;
%myloop;

/*AT THE END OF THE LOOP TRANSFER THE ENTIRE NEWS DATA INTO A PERMANENT DATA CALLED NEWS AND COMPUTE THE VARIANCES.

data news; set x1;
rename col1=PERMNO col2=YEARA col3=nr col4=ne;
data vdc.news; set news;
vr=nr**2;
ve=ne**2;
covre=nr*ne;
run;